Q: 10. [5 pts] An ideal gas undergoes the process at right. Is the magnitude of the heat exchanged with the gas greater than, less than, or equal to the magnitude of the work done on the gas?
A) Greater than
B) Less than
C) Equal to
D) Not enough information.


A: For an ideal gas,
$\Delta U=U_{f}-U_{i}=\frac{3}{2} P_{f} V_{f}-\frac{3}{2} P_{i} V_{i}=\frac{3}{2}(2 P)(V)-\frac{3}{2}(P)(3 V)=-\frac{3}{2} P V$
Meanwhile, the work $w$ is the area accumulated under the curve between points $i$ and $f$, i.e. the area shown in the figure to the right. This area can be calculated geometrically using the trapezoid area formula. Since the volume is decreasing, the work done by the system on the surroundings is negative. Therefore,
$w=-3 P V$
Now, we can use the first law of thermodynamics
$\Delta U=q-w$
Solving for $q$, and substituting $\Delta U$ and $w$, we have

$q=\Delta U+w=-\frac{3}{2} P V-3 P V=-\frac{9}{2} P V$
which is of greater magnitude than the work. There for the answer is $\underline{A}$.
(Note: the observation that $\Delta U<0$ alone proves that $\mathrm{w}>\mathrm{q}$. However, it does not imply that $|w|>|q|$ since it tells us nothing about whether $w$ and $q$ are positive or negative. That is why we must also calculate $w$ itself.)

